



Microwave Lab

Permittivity measurements

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Contact:

Ismael Vico Trivino: ismael.vicotrivino@epfl.ch

Mingxiang Gao: Mingxiang.gao@epfl.ch

1. Introduction

Dielectric materials strongly affect properties of many microwave devices, such as the losses and impedance of microstrip lines and coaxial cables, operating frequencies of resonators and antennas, absorber performance, etc. It is thus critical to be able to determine their characteristics, namely the relative permittivity and loss angle. In this exercise we study two (of many) methods of permittivity measurement.

2. Theoretical Background

2.1. Method of the minimum reflected power

Suppose that a rectangular waveguide, operating in the dominant mode (TE_{10}), is filled by a piece of lossless dielectric material of length L and terminated by a matched load. The reflected power is given by

$$P_{ref} = \frac{\frac{1}{4} \left(\frac{\lambda_g}{\lambda_{g\epsilon}} - \frac{\lambda_{g\epsilon}}{\lambda_g} \right)^2 \sin^2 \left(\frac{2\pi L}{\lambda_{g\epsilon}} \right)}{1 + \frac{1}{4} \left(\frac{\lambda_g}{\lambda_{g\epsilon}} - \frac{\lambda_{g\epsilon}}{\lambda_g} \right)^2 \sin^2 \left(\frac{2\pi L}{\lambda_{g\epsilon}} \right)} P_{inc}, \quad (1)$$

where λ_g and $\lambda_{g\epsilon}$ are the guide wavelengths in the empty and the dielectric filled waveguides, respectively. It can be noticed from (1) that the reflected power is minimum (zero) when

$$L = \frac{n\lambda_{g\epsilon}}{2}, \quad n \text{ being an integer.} \quad (2)$$

In the experiment, we search for the frequency where a minimum of the reflected power occurs. At that frequency, the equation (2) is satisfied. In addition, it is known that

$$\lambda_{g\epsilon} = \frac{\lambda}{\sqrt{\epsilon_r - \left(\frac{\lambda}{2a} \right)^2}}, \quad (3)$$

where a is the waveguide width and $\lambda = c/f$ is the free space wavelength. From (2) and (3) we obtain ϵ_r :

$$\epsilon_r = \left[\left(\frac{n}{2L} \right)^2 + \left(\frac{1}{2a} \right)^2 \right] \lambda^2. \quad (4)$$

In general, the relative permittivity has a real and imaginary part, the ratio of which is defined as a loss angle:

$$\epsilon_r = \epsilon' - j\epsilon'', \quad \tan \delta = \frac{\epsilon''}{\epsilon'}. \quad (5)$$

The method described above initially assumes a lossless dielectric, i.e. provides only ϵ' as if ϵ'' was equal to zero. Therefore, it is convenient for a preliminary estimation of relative permittivity of a given dielectric sample, but another method needs to be utilised to measure $\tan \delta$.

2.2. Method of the short-circuited waveguide

In the empty short-circuited waveguide, the minimums are located at $n\lambda_g/2$ from the short circuit plane. If we introduce a dielectric sample of length L against the short circuit, these minimums shift towards the short circuit as shown in Fig. 5.1. The shift Δl satisfies the following equation:

$$\frac{\tan x}{x} = \frac{\lambda_g}{2\pi L} \tan \left(\frac{2\pi(\Delta l + L)}{\lambda_g} \right) \quad (6)$$

The real part of the relative permittivity is given as

$$\epsilon' = \left(\frac{x\lambda}{2\pi L} \right)^2 + \left(\frac{\lambda}{\lambda_c} \right)^2 = \frac{\left(\frac{a}{\pi} \right)^2 \cdot \left(\frac{x}{L} \right)^2 + 1}{\left(\frac{2a}{\lambda_g} \right)^2 + 1}, \quad (7)$$

where $\lambda_c = 2a$ (the cutoff wavelength). Mathematically, the number of solutions in terms of x of (6) is infinite (in the Appendix, a table providing the values of the $\tan(x)/x$ function is given). Thus, the choice of x , necessary in order to calculate ϵ' from (7), is not unique. In other words, a 'guess' needs to be taken, and then, *a-posteriori*, the obtained value for the permittivity needs to be compared with the value obtained by a different method, such as the one described in the previous section. A coarse agreement of these values is typically enough to clearly confirm the right choice of x and the correct result obtained using this method.

Finally, the $\tan\delta$ can be calculated as

$$\tan\delta = \frac{\Delta x_1 - \Delta x}{\epsilon' L} \left(\frac{\lambda}{\lambda_g} \right)^2, \quad (8)$$

where Δx and Δx_1 , the distances between the 3 dB points (with respect to the minimum), are specified in Fig. 1. The positions of the minimums and the 3 dB points can be measured using the slotted section and SWR meter.

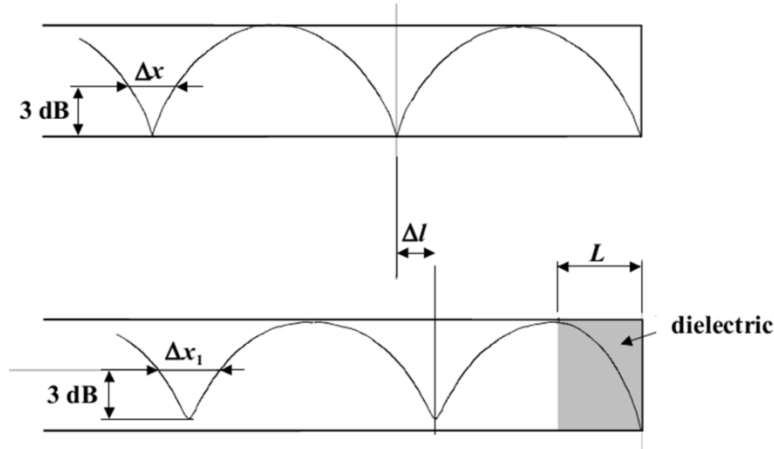


Fig. 1. Standing waves in the empty and filled waveguide terminated by the short circuit

3. Simulation software

In this lab, you will work with the simulation tool *ANSYS Electronics Desktop*. This software is broadly used by antenna and microwave engineers and allows to create 3D designs and run full-wave simulations which provide accurate results. You will use this software for this session and for the Antenna lab in the coming weeks.

In the following we describe some of the general features that can be useful for you to complete the exercise of this lab. Note that depending on the current version of the software the interface might be slightly different.

3.1. Ansys Electronics Desktop GUI

Let us start by the main graphical interface. There are six distinct sections/windows of interest within the Ansys Electronics Desktop user interface:

- The project tree, where you can see the active project and the different designs contained in it.
- The history tree, containing all the components of the design, and the history of actions taken to create them
- The 3D modeler window, where you can see the 3D representation of the components listed in History tree.
- The properties window, which lists the variables of the design, as well as specific properties of individual elements from the history tree, when selected.
- The message window, where the program logs information, warning and error messages.

The progress windows, where the system informs the user of the status of the simulations being run

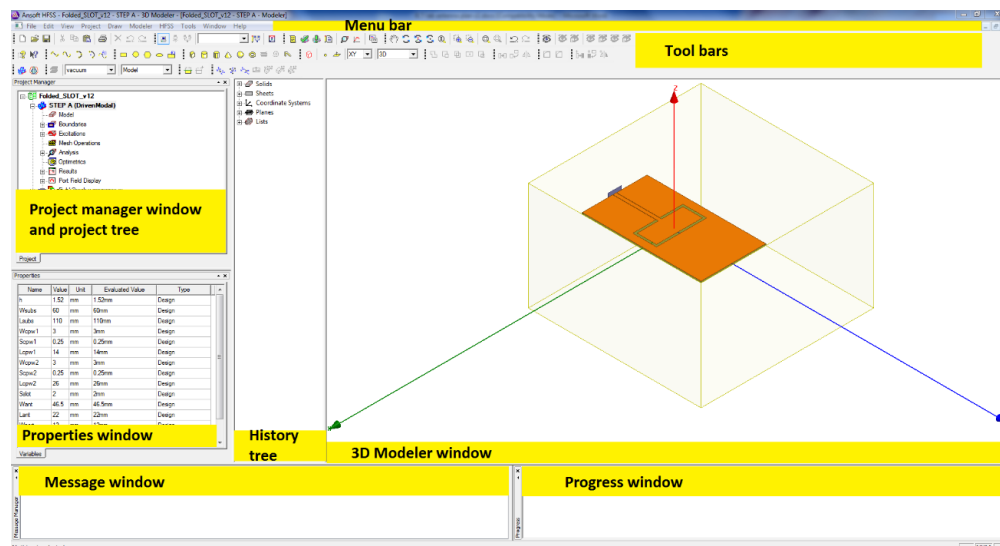


Fig. 2. Graphical interface of ANSYS Electronics Desktop

The largest window is the *3D modeler window*, where you can see the 3D model of the simulated antenna. You can change the view (zoom, rotate, etc...) using the icons highlighted in red in the following screenshot:

You can also access these buttons via shortcuts:

- Drag: SHIFT + Left Mouse Button
- Rotate model: Alt + Left Mouse Button
- Zoom in/out: Alt + SHIFT + Left Mouse Button



Fig. 3. Controls of the view of the 3D model

3.2. Change variable values

The simulator allows to parameterize the dimensions (among other things) of your design; this means that the dimensions are dynamic and you can modify them by just changing the numerical value of the corresponding variable. To access these variables, proceed as follows:

- Select the design in the Project manager (see picture) by left- clicking on its name.
- The list of properties is shown in the *properties window*.
- To modify a variable, you must modify its value in the corresponding column (Value). When introducing the number, pay attention to the units

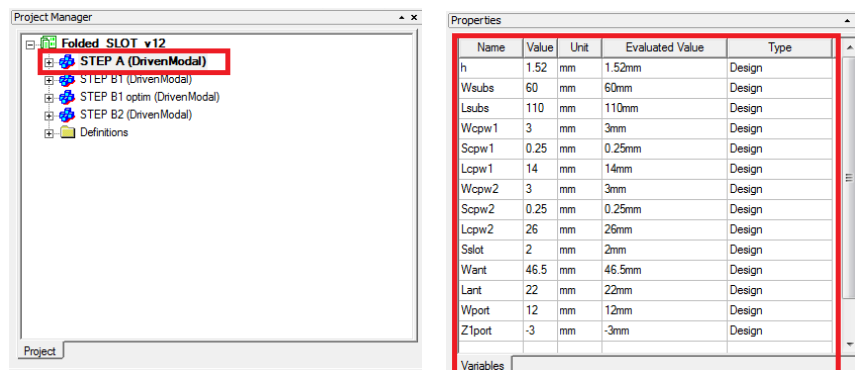


Fig. 4. Project tree (left) and variables of a design (right)

- Column “Evaluated Value” always shows the final value interpreted by *Ansys Electronics Desktop*.

3.3. Assignment of materials to a 3D object

In order to change the materials of an object under the “Solids” deployable list, you just need to right click on the object named “Sample”, and then on change materials.

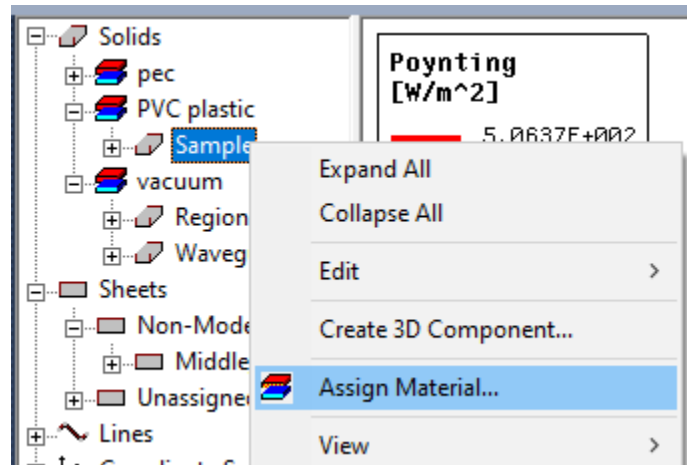


Fig. 5. Assignment of material to a 3D object in a design

3.4. Running simulations

Before running simulations, the design has to be completed, the excitation of the antenna defined, and the frequency of the simulation set-up (for this lab this has already been done).

To run the simulation, you must proceed as follows:

1. Ensure that the active design is the desired one, by left-clicking on its name in the Project Manager Window
2. To run the simulation, you can click on the Exclamation button (framed in green in the screenshot below); alternatively, you can right-click on Setup1 (framed in orange in the screenshot below) and select “Analyse”
3. The simulation will start directly, and you can follow its progress in the *Progress Window*.

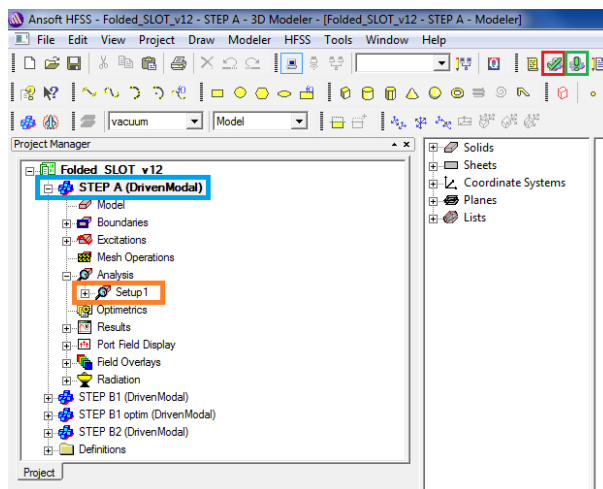


Fig. 6. Simulation settings

3.5. Results

Ansyz Electronics Desktop allows to plot the results of different antenna parameters. This is done in the branch “Results” from the *Project Manager* window.

For this lab session, the models are already given to you but you are expected to generate your own reports on the simulator and interpret the results. For example, if you need to generate a plot for the reflection coefficient of one port, you should right click on *Results* in the *Project Manager* window. Then you should move the mouse cursor on top of “Create Modal Solution Data Report” and then click on “Rectangular Plot”. There you can select the parameter that you want, which is already by default the reflection coefficient of port 1, $S(1,1)$, in dB.

To access them, you can left-click on *Results* in the *Project Manager* window. By left-clicking on the desired plot (double-click on the plot name or single click on the small icon on the left of the name), it will be displayed in the area marked with a green rectangle (see screenshot).

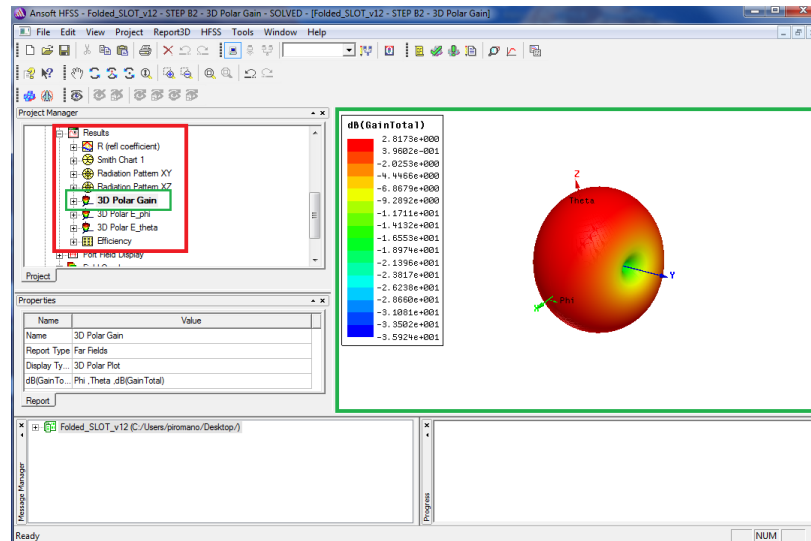


Fig. 7. Illustration on how to navigate through results

3.6. Save results

The specific plot can be easily copied to the clipboard by right-clicking on it and then selecting Copy Image. If you prefer to directly save the plot in image format, you can right-click and then select “Export...” and selecting the desired format (GIF, JPEG, BMP, TIFF), name and destination folder of the image file. You can also save the data of 2D plots in text format (CSV), by right clicking on the plot itself and on “Export...”.

4. Experiment in simulations

The measurement setup in Fig.8 is an example of the equipment needed to measure the permittivity of a dielectric sample in a laboratory. In this lab, you will work under the same principle than for the mentioned setup, but using a full wave simulator. You will use a 3D model of the rectangular waveguide, as depicted in Figure 10, that will allow you to visualize and understand how the propagation inside the wavelength occurs, and how the fields inside the waveguide change when one adds a dielectric sample. By properly measuring the change of the fields when there is a dielectric sample, one can calculate the relative permittivity of the material of which the sample is made.

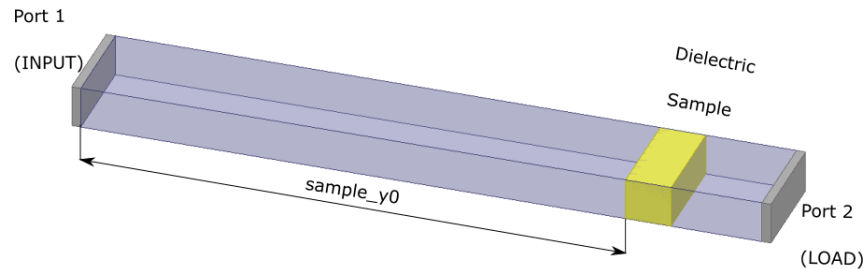


Fig. 8. 3D model of the rectangular waveguide that will be used during this lab.

4.1. Method of the minimum reflected power

In the models, we have already pre-defined a line and a plane at the centre of the rectangular waveguide. We can use these in ANSYS Electronics Desktop to visualize the standing waves formed inside the waveguide in 1D and 2D, respectively. These objects do not affect the simulation results and are only used for the visualization of the fields.

In this part, you are expected to generate the appropriate “Reports” in the simulator that allow you to process and interpret the results. You will need to plot and analyse the $S(1,1)$ for different samples made of various materials. Each time that you modify your model, you will need to re-run the simulations. Thus, it is highly recommended that you copy-paste your design to duplicate it under your project in order to keep all your results available throughout the session.

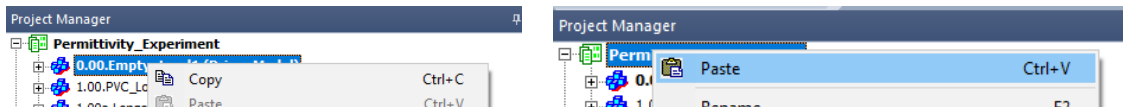


Fig. 9. How to duplicate the designs to keep the results when changing the material of the sample

Task 1: Generate the graph of the reflection coefficient for all the materials, and explain what is the difference between the results of an empty waveguide (when the sample is made of vacuum) and those of the dielectric samples.

Task 2: Before doing any calculation, and only by looking at the reflection coefficient results for all the materials, sort the samples from the lowest to the highest permittivity. Write your conclusions on Table 1 and justify your reasoning.

Task 3: Knowing the frequency f_0 of the minimum, the length L of each sample (the variable *sample_L* in the design), and the waveguide width ($a = 22.86$ mm for WR-90, the X-band waveguide), calculate the relative permittivity using (4). Write the results in Table 1 and repeat the measurement with the rest of the samples.

Note: the lengths of the samples are chosen so that we always observe the first resonance (when the sample is one half-wavelength long) in the X-band. Therefore, $n = 1$. The expected values of the relative permittivity for all the materials range between 2 and 4.

Material	f_0 , GHz	Order	ϵ_r
Amber			
Nylon			
Plexiglass			
Dellite			
PVC			

Table 1. Results of Tasks 1, 2 and 3.

4.2. Method of the short-circuited waveguide

Open the design *Part2.ShortCircuit_VACUUM*. In this design, the matched load has been changed by a short circuit, which will generate an ideal total reflection and create a standing wave pattern. The objective of this part of the lab is to show and analyse these standing waves to compute the permittivity of the dielectric samples.

In order to show the standing waves, go to *Sheets>Non-Model>Middle_Plane*. This plane has been defined as a non-model item in our design, and thus it does not affect the results and only serves as a support geometry on which we can plot the fields. Now, right click on *Middle_Plane*, then on *Plot Fields>E>ComplexMag_E*. On the window that appears, select *Setup1:Sweep* under the menu *Solution*. Now, choose the frequency at which you located a minimum of reflected power using the first method. Be careful to use the adequate frequency for each of the samples.

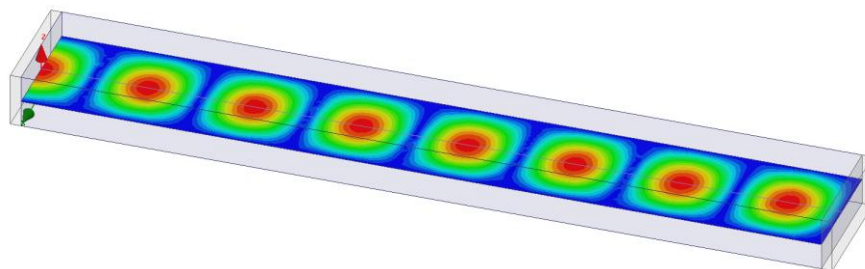


Fig. 10. Complex magnitude of the E field along the rectangular waveguide.

Now you should get something similar to what one can see on Fig. 12, where you can observe clearly the standing waves inside the rectangular waveguide. These plots are very useful to visualize how the waves propagate inside the structure. However, to perform more accurate calculations, you would need to generate a 2D plot. We pre-defined a line at the centre of the waveguide, where the standing waves are stronger. To show the E field along the line, you need to right click on “Results>Create Fields Report>Rectangular Plot”. There, you should select again *Setup1:Sweep*, and as a geometry you should use *Line*. Now select *ComplexMag_E* and dB under *Quantity* and *Function*, respectively, just as shown in Fig. 13.

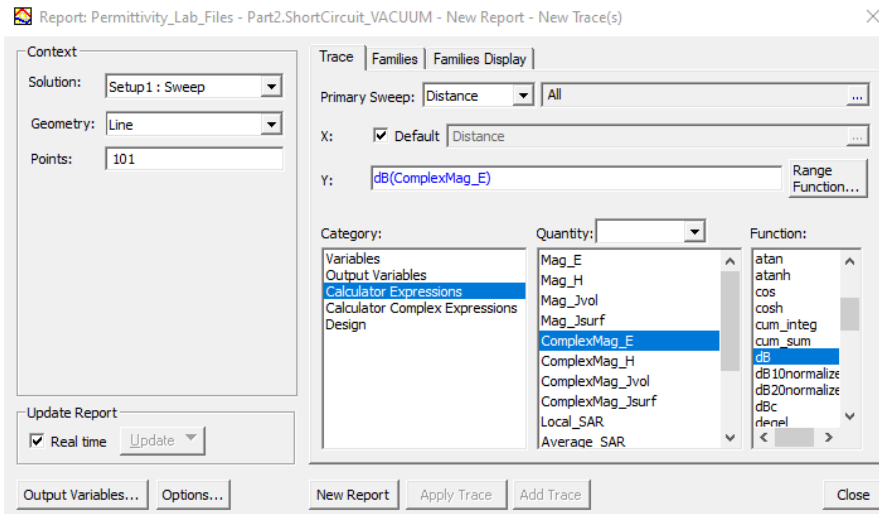


Fig. 11. Settings window to plot the complex magnitude of the E field (in dB) along a pre-defined line

Task 4: Plot the complex magnitude of the E field (in dB) along a the line for the designs *0.Part1.MatchedLoad_VACUUM* and *2.Part2.ShortCircuit_VACUUM*. You should see that the plots are very different. Explain the reason of these differences.

Task 5: Once you plotted the magnitude of the E field along a line, calculate or measure the guided wavelength λ_g at this frequency. Locate the position of a minimum (any of them) of the standing wave using the markers. This is the position p on the Table 2. Now, measure the physical distance between the two points which are at +3dB around the minimum. That distance is Δx on the Table 2.

At this point we want to simulate changing the material of the sample from vacuum to any of the materials we are working with. Therefore, right click on the *object Sample>Assign Materials* and select the corresponding material and run the simulation. According to Fig. 1, the minimums are shifted. Locate the position p_1 of the minimum the closest to the position p in the direction of the load and calculate $\Delta l = p - p_1$. Measure Δx_1 in the same manner as you did for Δx . From these values compute ϵ' and $\tan \delta$ using (6)-(8). Compare the obtained value for ϵ' and ϵ' obtained by the first method. A reasonable agreement between the two results is the confirmation that a good value for the variable x has been chosen.

Task 6: Repeat the procedure for the remaining samples and complete the Table 2.

Sample	f_0 , GHz	λ , mm	λ_g , mm	L , mm	p , mm	p_1 , mm	Δl , mm	$\frac{\tan x}{x}$	x	Δx , mm	Δx_1 , mm	ϵ'	$\tan \delta$
Amber													
Nylon													
Plexig.													
Dellite													
PVC													

Table 2. Measurement results by using the method of the short-circuited waveguide.

5. Experiment in the lab

In this part of the lab, you are asked to reproduce the measurements done in simulations with a measurement setup in the lab.

Note: The experiment in the lab is essentially the same as in simulations, and thus you may find that part of the explanation is repeated.

5.1. Utilised equipment

- Sweep generator *HP 8350* with the RF plug-in *HP86250D*
- Oscilloscope *Textronix 5103N*
- SWR meter *HP 415E*
- Coaxial to waveguide adapter *H&S 3102*
- Ferrite isolator *Sivers Lab PM 7041X*
- Wavemeter *HP X532B*
- Variable attenuator *HP X382A*
- Directional coupler 3 dB *HP X752A*
- Fixed 10 dB attenuator
- Crystal detector *HP X424A*
- Slotted section *HP X809C* with the detector probe *HP 444A* and a precision scale
- Waveguide support for dielectric samples
- Waveguide termination (matched load)
- Waveguide short
- 5 samples of dielectric materials: amber, Nylon, Plexiglass, Dellite, and PVC
- Calliper

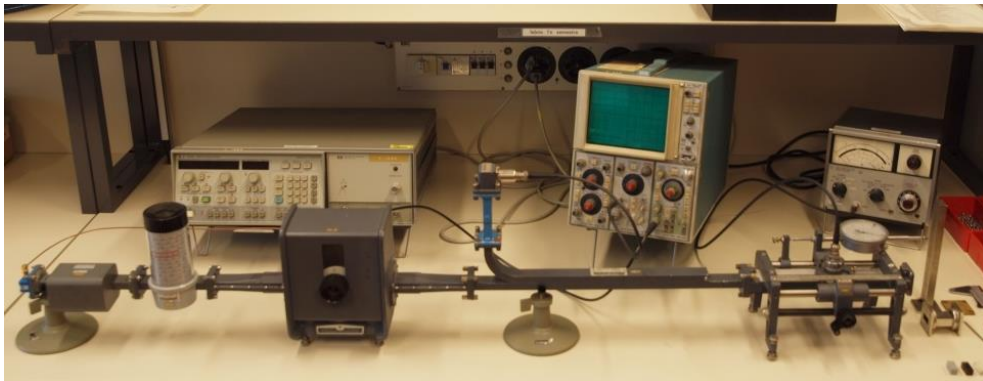


Fig. 12. Setup for the measurement of permittivity

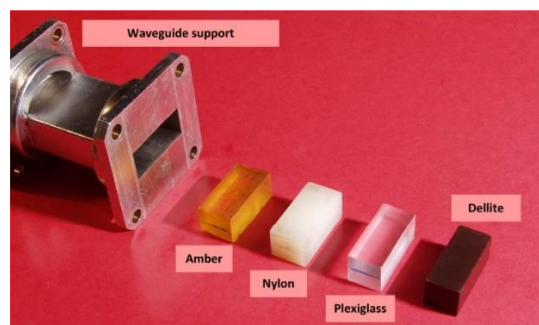


Fig. 13. Short piece of rectangular wavelength and dielectric samples

5.2. Experiment setup

The experiment setup shown in Fig. 9 allows permittivity measurements by both described methods. The system is fed by a microwave signal from the sweep generator via the coaxial-to-waveguide adapter. The power level can be regulated owing to an internal attenuator of the generator. The generator also provides a saw-tooth signal (synchronised with the swept microwave frequency) that is driving the x-plates of the oscilloscope. The ferrite isolator protects the generator from potential reflections. The wavemeter enables precise frequency measurements. The precise variable attenuator is utilised for determining the 3 dB points from the minima (in the method of the short-circuited waveguide). The 3-dB directional coupler with the mounted detector whose output is connected to the oscilloscope (y-plates) enables the visualisation of the reflected signal in the first method. The slotted section with the detector probe and the SWR meter are used to find the critical points of the standing wave (minima and 3 dB-points). The samples are placed in the waveguide support, which is essentially a small straight waveguide section. The matched load and waveguide short complete the setup.

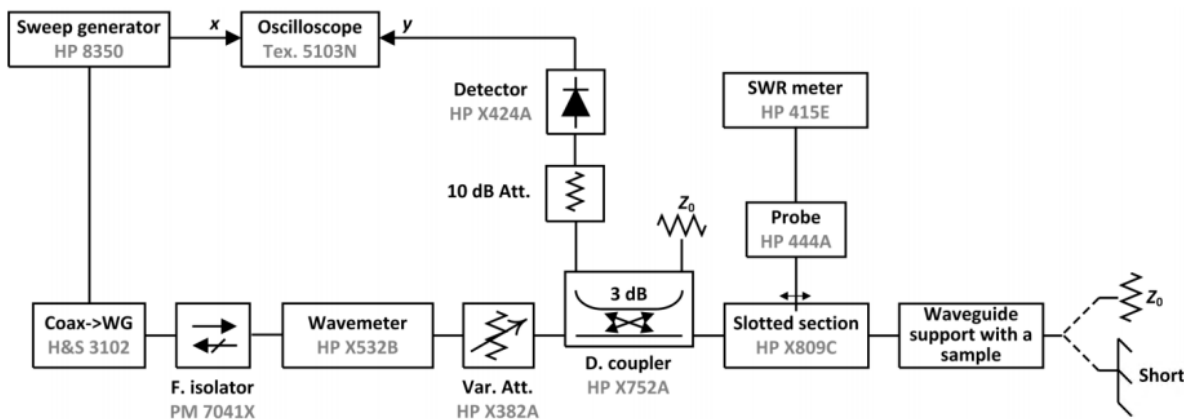


Fig. 14. Experiment setup for the permittivity measurements

5.3. Measurements

5.3.1. Method of the minimum reflected power

Experiment setup preparation

Turn the sweep generator on, set the frequency sweep between 8 GHz and 12 GHz, the sweep velocity to 0.02 s, no modulation, RF blank, 'ALC' to int. Set the power to the maximum, but verify that the LED indicating unlevelled signal is off. Turn on the oscilloscope (normal mode), set it to X-Y function (the x-axis, controlled by the generator, represents frequency, while the y-axis is the detected signal, therefore proportional to the reflected power). Take the probe out from the slotted section. Set the variable attenuator to 0 dB.

Note: this time there is no need to check if the detector is in the quadratic zone, because we will only use it to indicate the minima. However, utilise the RF on/off button on the generator to avoid unnecessary radiation while mounting/dismounting waveguide components.

Measurement

Place the first sample (amber) into the waveguide support (not into the matched load or the slotted section) and terminate it in the matched load. Adjust the sensitivity of the oscilloscope and the width of the frequency sweep to obtain a trace covering the entire screen, with a clearly indicated minimum. To measure the frequency f_0 of this minimum, we can:

a) Use directly the dip caused by the wavemeter if it is clearly visible in the minimum (which will be the case only with lossy dielectrics);

b) Set the generator in 'CW' mode and precisely adjust the minimum by looking at the oscilloscope screen. This can be more precisely done using the SWR meter. Connect this instrument to the detector instead of the oscilloscope (turn on the 1 kHz-modulation on the generator) and adjust the frequency to reach the minimum. Then, measure this frequency using the wavemeter.

Knowing the frequency f_0 of the minimum, the length L of each sample, and the waveguide width ($a = 22.86$ mm for WR-90, the X-band waveguide), calculate the relative permittivity using (4). Write the results in Table 5.1 and repeat the measurement with the rest of the samples.

Note: the lengths of the samples are chosen so that we always observe the first resonance (when the sample is one half-wavelength long) in the X-band. Therefore, $n = 1$

5.3.2. Method of the short-circuited waveguide

Experiment setup preparation

Turn-off the oscilloscope. Remove the samples from the waveguide support and terminate it by the short circuit. Set the generator in the 'CW' mode, turn on the 1 kHz-modulation, put the detector probe into the slotted section (probe depth should be roughly 0.5 mm) and connect it to the SWR meter. Turn the power level to maximum (but still levelled). Set the variable attenuator to 0 dB.

Measurement

Adjust the frequency to the value f_0 where the minimum was found in the first method (for the first sample). Calculate the free-space wavelength λ at this frequency. Calculate or measure the guided wavelength λ_g at this frequency. Place the probe precisely into a minimum, preferably the one that is roughly in the middle of the slotted section. This is the position p . Adjust the needle of the SWR meter to some round value in the middle of the scale. Now, set the variable attenuator to 3 dB and move the probe to the points (on each side of the minimum) where the SWR meter needle gets back to the previously adjusted value. Measure the physical distance between these two points Δx . Introduce the first dielectric sample into the waveguide support and terminate it by the short

circuit. It is very important that the sample is in the contact with the shorting wall. According to Fig. 1, the minima are shifted. Measure the position p_1 of the closest minimum to the position p in the direction of the load and calculate $\Delta l = p - p_1$. Measure Δx_1 in the same manner as above. From these values compute ϵ' and $\tan\delta$ using (6)-(8). Compare the obtained value for ϵ' with the value for ϵ' obtained by the first method. A reasonable agreement between the two results is the confirmation that a good value for the variable x has been chosen.

Task 7: Validate that the results from the measurement are coherent with the results obtained in simulations

6. References

- [1] Max Sucher and Jerome Fox (editors), *Handbook of microwave measurements (Volume II)*, 3rd edition, Polytechnic Press Brooklyn, 1963.
- [2] David Pozar, *Microwave Engineering*, 4th edition, John Wiley & Sons, 2011.

7. Appendix

7.1. Function $\tan(x)/x$

x	$\tan(x)/x$	x	$\tan(x)/x$	x	$\tan(x)/x$	x	$\tan(x)/x$
0.00	1.0000	0.43	1.0666	0.86	1.3506	1.29	2.6878
0.01	1.0000	0.44	1.0700	0.87	1.3624	1.30	2.7708
0.02	1.0001	0.45	1.0735	0.88	1.3746	1.31	2.8604
0.03	1.0003	0.46	1.0771	0.89	1.3872	1.32	2.9571
0.04	1.0005	0.47	1.0808	0.90	1.4002	1.33	3.0619
0.05	1.0008	0.48	1.0846	0.91	1.4136	1.34	3.1758
0.06	1.0012	0.49	1.0885	0.92	1.4275	1.35	3.3002
0.07	1.0016	0.50	1.0926	0.93	1.4418	1.36	3.4364
0.08	1.0021	0.51	1.0968	0.94	1.4566	1.37	3.5862
0.09	1.0027	0.52	1.1011	0.95	1.4720	1.38	3.7518
0.10	1.0033	0.53	1.1055	0.96	1.4879	1.39	3.9357
0.11	1.0041	0.54	1.1101	0.97	1.5043	1.40	4.1413
0.12	1.0048	0.55	1.1147	0.98	1.5214	1.41	4.3726
0.13	1.0057	0.56	1.1196	0.99	1.5391	1.42	4.6346
0.14	1.0066	0.57	1.1245	1.00	1.5574	1.43	4.9339
0.15	1.0076	0.58	1.1296	1.01	1.5764	1.44	5.2790
0.16	1.0086	0.59	1.1348	1.02	1.5962	1.45	5.6814
0.17	1.0097	0.60	1.1402	1.03	1.6167	1.46	6.1566
0.18	1.0109	0.61	1.1458	1.04	1.6381	1.47	6.7261
0.19	1.0122	0.62	1.1515	1.05	1.6603	1.48	7.4212
0.20	1.0136	0.63	1.1573	1.06	1.6834	1.49	8.2885
0.21	1.0150	0.64	1.1633	1.07	1.7075	1.50	9.4009
0.22	1.0165	0.65	1.1695	1.08	1.7326	1.51	10.8795
0.23	1.0180	0.66	1.1759	1.09	1.7588	1.52	12.9405
0.24	1.0197	0.67	1.1825	1.10	1.7861	1.53	16.0120
0.25	1.0214	0.68	1.1892	1.11	1.8147	1.54	21.0787
0.26	1.0232	0.69	1.1961	1.12	1.8446	1.55	31.0184
0.27	1.0250	0.70	1.2033	1.13	1.8759	1.56	59.3721
0.28	1.0270	0.71	1.2106	1.14	1.9087	1.57	799.8507
0.29	1.0290	0.72	1.2181	1.15	1.9430	1.58	-68.7653
0.30	1.0311	0.73	1.2259	1.16	1.9791	1.59	-32.7465
0.31	1.0333	0.74	1.2339	1.17	2.0171	1.60	-21.3953
0.32	1.0356	0.75	1.2421	1.18	2.0570	1.61	-15.8352
0.33	1.0380	0.76	1.2506	1.19	2.0991	1.62	-12.5354
0.34	1.0404	0.77	1.2593	1.20	2.1435	1.63	-10.3504
0.35	1.0429	0.78	1.2683	1.21	2.1904	1.64	-8.7970
0.36	1.0456	0.79	1.2775	1.22	2.2400	1.65	-7.6359

0.37	1.0483	0.80	1.2870	1.23	2.2925	1.66	-6.7353
0.38	1.0511	0.81	1.2969	1.24	2.3483	1.67	-6.0163
0.39	1.0540	0.82	1.3070	1.25	2.4077	1.68	-5.4290
0.40	1.0570	0.83	1.3174	1.26	2.4708	1.69	-4.9404
0.41	1.0601	0.84	1.3281	1.27	2.5383	1.70	-4.5274
0.42	1.0633	0.85	1.3392	1.28	2.6104	1.71	-4.1738
1.72	-3.8677	2.22	-0.5935	2.72	-0.1649	3.22	0.0244
1.73	-3.6001	2.23	-0.5787	2.73	-0.1599	3.23	0.0274
1.74	-3.3641	2.24	-0.5644	2.74	-0.1550	3.24	0.0305
1.75	-3.1545	2.25	-0.5505	2.75	-0.1502	3.25	0.0335
1.76	-2.9671	2.26	-0.5370	2.76	-0.1454	3.26	0.0365
1.77	-2.7985	2.27	-0.5239	2.77	-0.1407	3.27	0.0395
1.78	-2.6461	2.28	-0.5111	2.78	-0.1361	3.28	0.0425
1.79	-2.5076	2.29	-0.4987	2.79	-0.1315	3.29	0.0454
1.80	-2.3813	2.30	-0.4866	2.80	-0.1270	3.30	0.0484
1.81	-2.2655	2.31	-0.4749	2.81	-0.1225	3.31	0.0514
1.82	-2.1590	2.32	-0.4634	2.82	-0.1181	3.32	0.0543
1.83	-2.0608	2.33	-0.4523	2.83	-0.1138	3.33	0.0573
1.84	-1.9698	2.34	-0.4414	2.84	-0.1095	3.34	0.0602
1.85	-1.8854	2.35	-0.4308	2.85	-0.1053	3.35	0.0631
1.86	-1.8069	2.36	-0.4205	2.86	-0.1011	3.36	0.0661
1.87	-1.7336	2.37	-0.4104	2.87	-0.0970	3.37	0.0690
1.88	-1.6651	2.38	-0.4006	2.88	-0.0930	3.38	0.0719
1.89	-1.6009	2.39	-0.3910	2.89	-0.0889	3.39	0.0748
1.90	-1.5406	2.40	-0.3817	2.90	-0.0850	3.40	0.0777
1.91	-1.4838	2.41	-0.3725	2.91	-0.0810	3.41	0.0807
1.92	-1.4304	2.42	-0.3636	2.92	-0.0772	3.42	0.0836
1.93	-1.3799	2.43	-0.3549	2.93	-0.0733	3.43	0.0865
1.94	-1.3321	2.44	-0.3463	2.94	-0.0695	3.44	0.0894
1.95	-1.2869	2.45	-0.3380	2.95	-0.0658	3.45	0.0923
1.96	-1.2440	2.46	-0.3298	2.96	-0.0620	3.46	0.0953
1.97	-1.2033	2.47	-0.3218	2.97	-0.0583	3.47	0.0982
1.98	-1.1646	2.48	-0.3140	2.98	-0.0547	3.48	0.1011
1.99	-1.1277	2.49	-0.3063	2.99	-0.0511	3.49	0.1041
2.00	-1.0925	2.50	-0.2988	3.00	-0.0475	3.50	0.1070
2.01	-1.0590	2.51	-0.2915	3.01	-0.0440	3.51	0.1100
2.02	-1.0269	2.52	-0.2843	3.02	-0.0405	3.52	0.1129
2.03	-0.9963	2.53	-0.2772	3.03	-0.0370	3.53	0.1159
2.04	-0.9669	2.54	-0.2703	3.04	-0.0335	3.54	0.1189
2.05	-0.9388	2.55	-0.2635	3.05	-0.0301	3.55	0.1219
2.06	-0.9118	2.56	-0.2568	3.06	-0.0267	3.56	0.1249
2.07	-0.8860	2.57	-0.2503	3.07	-0.0234	3.57	0.1279
2.08	-0.8611	2.58	-0.2439	3.08	-0.0200	3.58	0.1310
2.09	-0.8372	2.59	-0.2376	3.09	-0.0167	3.59	0.1340
2.10	-0.8142	2.60	-0.2314	3.10	-0.0134	3.60	0.1371
2.11	-0.7921	2.61	-0.2253	3.11	-0.0102	3.61	0.1402

2.12	-0.7707	2.62	-0.2193	3.12	-0.0069	3.62	0.1433
2.13	-0.7502	2.63	-0.2135	3.13	-0.0037	3.63	0.1464
2.14	-0.7303	2.64	-0.2077	3.14	-0.0005	3.64	0.1495
2.15	-0.7112	2.65	-0.2021	3.15	0.0027	3.65	0.1527
2.16	-0.6926	2.66	-0.1965	3.16	0.0058	3.66	0.1559
2.17	-0.6747	2.67	-0.1910	3.17	0.0090	3.67	0.1591
2.18	-0.6574	2.68	-0.1856	3.18	0.0121	3.68	0.1623
2.19	-0.6407	2.69	-0.1803	3.19	0.0152	3.69	0.1656
2.20	-0.6245	2.70	-0.1751	3.20	0.0183	3.70	0.1688
2.21	-0.6088	2.71	-0.1699	3.21	0.0213	3.71	0.1722
3.72	0.1755	4.22	0.4417	4.72	-27.8360	5.22	-0.3444
3.73	0.1789	4.23	0.4515	4.73	-12.0035	5.23	-0.3358
3.74	0.1823	4.24	0.4616	4.74	-7.6389	5.24	-0.3275
3.75	0.1857	4.25	0.4721	4.75	-5.5948	5.25	-0.3195
3.76	0.1892	4.26	0.4830	4.76	-4.4092	5.26	-0.3118
3.77	0.1928	4.27	0.4944	4.77	-3.6349	5.27	-0.3043
3.78	0.1963	4.28	0.5063	4.78	-3.0895	5.28	-0.2970
3.79	0.1999	4.29	0.5186	4.79	-2.6845	5.29	-0.2900
3.80	0.2036	4.30	0.5316	4.80	-2.3718	5.30	-0.2833
3.81	0.2073	4.31	0.5451	4.81	-2.1231	5.31	-0.2767
3.82	0.2110	4.32	0.5593	4.82	-1.9205	5.32	-0.2703
3.83	0.2148	4.33	0.5742	4.83	-1.7522	5.33	-0.2641
3.84	0.2186	4.34	0.5899	4.84	-1.6103	5.34	-0.2581
3.85	0.2225	4.35	0.6063	4.85	-1.4889	5.35	-0.2523
3.86	0.2265	4.36	0.6237	4.86	-1.3838	5.36	-0.2466
3.87	0.2305	4.37	0.6420	4.87	-1.2920	5.37	-0.2411
3.88	0.2346	4.38	0.6614	4.88	-1.2111	5.38	-0.2358
3.89	0.2387	4.39	0.6819	4.89	-1.1393	5.39	-0.2306
3.90	0.2429	4.40	0.7037	4.90	-1.0750	5.40	-0.2255
3.91	0.2472	4.41	0.7269	4.91	-1.0172	5.41	-0.2205
3.92	0.2516	4.42	0.7516	4.92	-0.9649	5.42	-0.2157
3.93	0.2560	4.43	0.7780	4.93	-0.9174	5.43	-0.2110
3.94	0.2605	4.44	0.8063	4.94	-0.8740	5.44	-0.2064
3.95	0.2651	4.45	0.8367	4.95	-0.8342	5.45	-0.2019
3.96	0.2698	4.46	0.8694	4.96	-0.7975	5.46	-0.1975
3.97	0.2745	4.47	0.9048	4.97	-0.7637	5.47	-0.1933
3.98	0.2794	4.48	0.9432	4.98	-0.7324	5.48	-0.1891
3.99	0.2844	4.49	0.9849	4.99	-0.7032	5.49	-0.1850
4.00	0.2895	4.50	1.0305	5.00	-0.6761	5.50	-0.1810
4.01	0.2946	4.51	1.0806	5.01	-0.6508	5.51	-0.1771
4.02	0.2999	4.52	1.1357	5.02	-0.6270	5.52	-0.1733
4.03	0.3054	4.53	1.1969	5.03	-0.6048	5.53	-0.1695
4.04	0.3109	4.54	1.2650	5.04	-0.5838	5.54	-0.1659
4.05	0.3166	4.55	1.3415	5.05	-0.5641	5.55	-0.1623
4.06	0.3224	4.56	1.4279	5.06	-0.5454	5.56	-0.1588
4.07	0.3284	4.57	1.5264	5.07	-0.5278	5.57	-0.1553

4.08	0.3345	4.58	1.6396	5.08	-0.5111	5.58	-0.1519
4.09	0.3408	4.59	1.7712	5.09	-0.4953	5.59	-0.1486
4.10	0.3472	4.60	1.9261	5.10	-0.4803	5.60	-0.1453
4.11	0.3538	4.61	2.1112	5.11	-0.4660	5.61	-0.1421
4.12	0.3606	4.62	2.3361	5.12	-0.4523	5.62	-0.1390
4.13	0.3677	4.63	2.6156	5.13	-0.4393	5.63	-0.1359
4.14	0.3749	4.64	2.9720	5.14	-0.4269	5.64	-0.1329
4.15	0.3823	4.65	3.4425	5.15	-0.4150	5.65	-0.1299
4.16	0.3900	4.66	4.0924	5.16	-0.4037	5.66	-0.1270
4.17	0.3979	4.67	5.0486	5.17	-0.3928	5.67	-0.1241
4.18	0.4061	4.68	6.5949	5.18	-0.3823	5.68	-0.1213
4.19	0.4145	4.69	9.5218	5.19	-0.3723	5.69	-0.1185
4.20	0.4233	4.70	17.1729	5.20	-0.3626	5.70	-0.1157
4.21	0.4323	4.71	88.8721	5.21	-0.3533	5.71	-0.1130
5.72	-0.1104	6.22	-0.0102	6.72	0.0695	7.22	0.1884
5.73	-0.1078	6.23	-0.0085	6.73	0.0712	7.23	0.1921
5.74	-0.1052	6.24	-0.0069	6.74	0.0729	7.24	0.1960
5.75	-0.1026	6.25	-0.0053	6.75	0.0747	7.25	0.1999
5.76	-0.1001	6.26	-0.0037	6.76	0.0764	7.26	0.2040
5.77	-0.0977	6.27	-0.0021	6.77	0.0782	7.27	0.2081
5.78	-0.0952	6.28	-0.0005	6.78	0.0800	7.28	0.2124
5.79	-0.0928	6.29	0.0011	6.79	0.0818	7.29	0.2169
5.80	-0.0905	6.30	0.0027	6.80	0.0836	7.30	0.2214
5.81	-0.0881	6.31	0.0043	6.81	0.0854	7.31	0.2262
5.82	-0.0858	6.32	0.0058	6.82	0.0873	7.32	0.2310
5.83	-0.0835	6.33	0.0074	6.83	0.0891	7.33	0.2361
5.84	-0.0813	6.34	0.0090	6.84	0.0910	7.34	0.2413
5.85	-0.0791	6.35	0.0105	6.85	0.0929	7.35	0.2467
5.86	-0.0769	6.36	0.0121	6.86	0.0948	7.36	0.2523
5.87	-0.0747	6.37	0.0137	6.87	0.0968	7.37	0.2581
5.88	-0.0725	6.38	0.0152	6.88	0.0988	7.38	0.2641
5.89	-0.0704	6.39	0.0168	6.89	0.1008	7.39	0.2704
5.90	-0.0683	6.40	0.0183	6.90	0.1028	7.40	0.2769
5.91	-0.0662	6.41	0.0199	6.91	0.1048	7.41	0.2837
5.92	-0.0642	6.42	0.0214	6.92	0.1069	7.42	0.2908
5.93	-0.0622	6.43	0.0230	6.93	0.1090	7.43	0.2982
5.94	-0.0602	6.44	0.0246	6.94	0.1111	7.44	0.3059
5.95	-0.0582	6.45	0.0261	6.95	0.1132	7.45	0.3140
5.96	-0.0562	6.46	0.0277	6.96	0.1154	7.46	0.3225
5.97	-0.0542	6.47	0.0292	6.97	0.1176	7.47	0.3313
5.98	-0.0523	6.48	0.0308	6.98	0.1199	7.48	0.3407
5.99	-0.0504	6.49	0.0323	6.99	0.1222	7.49	0.3505
6.00	-0.0485	6.50	0.0339	7.00	0.1245	7.50	0.3608
6.01	-0.0466	6.51	0.0355	7.01	0.1268	7.51	0.3717
6.02	-0.0448	6.52	0.0370	7.02	0.1292	7.52	0.3832
6.03	-0.0429	6.53	0.0386	7.03	0.1317	7.53	0.3955

6.04	-0.0411	6.54	0.0402	7.04	0.1341	7.54	0.4084
6.05	-0.0393	6.55	0.0417	7.05	0.1367	7.55	0.4222
6.06	-0.0375	6.56	0.0433	7.06	0.1392	7.56	0.4369
6.07	-0.0357	6.57	0.0449	7.07	0.1418	7.57	0.4526
6.08	-0.0339	6.58	0.0465	7.08	0.1445	7.58	0.4694
6.09	-0.0321	6.59	0.0481	7.09	0.1472	7.59	0.4874
6.10	-0.0304	6.60	0.0497	7.10	0.1500	7.60	0.5069
6.11	-0.0286	6.61	0.0513	7.11	0.1528	7.61	0.5279
6.12	-0.0269	6.62	0.0529	7.12	0.1557	7.62	0.5506
6.13	-0.0252	6.63	0.0545	7.13	0.1586	7.63	0.5753
6.14	-0.0235	6.64	0.0561	7.14	0.1616	7.64	0.6023
6.15	-0.0218	6.65	0.0578	7.15	0.1647	7.65	0.6319
6.16	-0.0201	6.66	0.0594	7.16	0.1679	7.66	0.6645
6.17	-0.0184	6.67	0.0611	7.17	0.1711	7.67	0.7006
6.18	-0.0168	6.68	0.0627	7.18	0.1744	7.68	0.7408
6.19	-0.0151	6.69	0.0644	7.19	0.1777	7.69	0.7859
6.20	-0.0134	6.70	0.0661	7.20	0.1812	7.70	0.8367
6.21	-0.0118	6.71	0.0678	7.21	0.1847	7.71	0.8946
7.72	0.9610	8.22	-0.3174	8.72	-0.0975	9.22	-0.0225
7.73	1.0381	8.23	-0.3078	8.73	-0.0955	9.23	-0.0214
7.74	1.1286	8.24	-0.2986	8.74	-0.0934	9.24	-0.0202
7.75	1.2364	8.25	-0.2899	8.75	-0.0914	9.25	-0.0191
7.76	1.3671	8.26	-0.2816	8.76	-0.0895	9.26	-0.0180
7.77	1.5289	8.27	-0.2737	8.77	-0.0875	9.27	-0.0168
7.78	1.7342	8.28	-0.2661	8.78	-0.0856	9.28	-0.0157
7.79	2.0036	8.29	-0.2589	8.79	-0.0838	9.29	-0.0146
7.80	2.3727	8.30	-0.2520	8.80	-0.0819	9.30	-0.0135
7.81	2.9094	8.31	-0.2453	8.81	-0.0801	9.31	-0.0124
7.82	3.7617	8.32	-0.2390	8.82	-0.0784	9.32	-0.0113
7.83	5.3245	8.33	-0.2328	8.83	-0.0766	9.33	-0.0102
7.84	9.1222	8.34	-0.2270	8.84	-0.0749	9.34	-0.0091
7.85	31.9939	8.35	-0.2213	8.85	-0.0732	9.35	-0.0080
7.86	-21.1394	8.36	-0.2159	8.86	-0.0715	9.36	-0.0069
7.87	-7.9318	8.37	-0.2106	8.87	-0.0699	9.37	-0.0059
7.88	-4.8764	8.38	-0.2055	8.88	-0.0682	9.38	-0.0048
7.89	-3.5173	8.39	-0.2006	8.89	-0.0666	9.39	-0.0037
7.90	-2.7487	8.40	-0.1959	8.90	-0.0650	9.40	-0.0026
7.91	-2.2544	8.41	-0.1913	8.91	-0.0635	9.41	-0.0016
7.92	-1.9098	8.42	-0.1869	8.92	-0.0619	9.42	-0.0005
7.93	-1.6557	8.43	-0.1826	8.93	-0.0604	9.43	0.0006
7.94	-1.4605	8.44	-0.1785	8.94	-0.0589	9.44	0.0016
7.95	-1.3060	8.45	-0.1745	8.95	-0.0574	9.45	0.0027
7.96	-1.1805	8.46	-0.1706	8.96	-0.0560	9.46	0.0037
7.97	-1.0766	8.47	-0.1668	8.97	-0.0545	9.47	0.0048
7.98	-0.9891	8.48	-0.1631	8.98	-0.0531	9.48	0.0058
7.99	-0.9145	8.49	-0.1595	8.99	-0.0517	9.49	0.0069

8.00	-0.8500	8.50	-0.1560	9.00	-0.0503	9.50	0.0079
8.01	-0.7937	8.51	-0.1527	9.01	-0.0489	9.51	0.0090
8.02	-0.7441	8.52	-0.1494	9.02	-0.0475	9.52	0.0100
8.03	-0.7002	8.53	-0.1462	9.03	-0.0461	9.53	0.0111
8.04	-0.6609	8.54	-0.1430	9.04	-0.0448	9.54	0.0121
8.05	-0.6256	8.55	-0.1400	9.05	-0.0435	9.55	0.0132
8.06	-0.5937	8.56	-0.1370	9.06	-0.0421	9.56	0.0142
8.07	-0.5647	8.57	-0.1341	9.07	-0.0408	9.57	0.0153
8.08	-0.5382	8.58	-0.1313	9.08	-0.0396	9.58	0.0163
8.09	-0.5140	8.59	-0.1285	9.09	-0.0383	9.59	0.0174
8.10	-0.4917	8.60	-0.1258	9.10	-0.0370	9.60	0.0184
8.11	-0.4711	8.61	-0.1232	9.11	-0.0357	9.61	0.0195
8.12	-0.4520	8.62	-0.1206	9.12	-0.0345	9.62	0.0206
8.13	-0.4343	8.63	-0.1181	9.13	-0.0333	9.63	0.0216
8.14	-0.4177	8.64	-0.1156	9.14	-0.0320	9.64	0.0227
8.15	-0.4023	8.65	-0.1132	9.15	-0.0308	9.65	0.0237
8.16	-0.3879	8.66	-0.1108	9.16	-0.0296	9.66	0.0248
8.17	-0.3743	8.67	-0.1085	9.17	-0.0284	9.67	0.0259
8.18	-0.3616	8.68	-0.1062	9.18	-0.0272	9.68	0.0270
8.19	-0.3496	8.69	-0.1040	9.19	-0.0260	9.69	0.0280
8.20	-0.3383	8.70	-0.1018	9.20	-0.0249	9.70	0.0291
8.21	-0.3275	8.71	-0.0996	9.21	-0.0237	9.71	0.0302
9.72	0.0313						
9.73	0.0324						
9.74	0.0335						
9.75	0.0346						
9.76	0.0357						
9.77	0.0368						
9.78	0.0379						
9.79	0.0391						
9.80	0.0402						
9.81	0.0413						
9.82	0.0425						
9.83	0.0436						
9.84	0.0448						
9.85	0.0460						
9.86	0.0472						
9.87	0.0483						
9.88	0.0495						
9.89	0.0508						
9.90	0.0520						
9.91	0.0532						
9.92	0.0544						
9.93	0.0557						
9.94	0.0570						
9.95	0.0582						

9.96	0.0595
9.97	0.0608
9.98	0.0622
9.99	0.0635
10.00	0.0648